CloudSat Project

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Level 2B Radar-Visible Optical Depth Cloud Water Content (2B-CWC-RVOD) Process Description Document

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1 Introduction

The CloudSat Radar-Visible Optical Depth Cloud Water Content Product (2B-CWC-RVOD) contains retrieved estimates of cloud liquid and ice water content, effective radius, and related quantities for each radar profile measured by the Cloud Profiling Radar on CloudSat. Retrievals are performed separately for the liquid and ice phases; the two sets of results are then combined in a simple way to obtain a composite profile that is consistent with the input measurements.

This radar-visible optical depth product uses a combination of measured radar reflectivity factor (from the CloudSat 2B-GEOPROF product) together with estimates of visible optical dpeth (from the CloudSat 2B-TAU product) to constrain the cloud retrievals more tightly than in the radar-only product (2B-CWC-RO) and presumably yielding more accurate results. However, retrievals of visible optical depth are difficult or impossible in many cases, due to the complexity of the targets and the simplifying assumptions made necessary by the data volumes associated with an operational satellite. Rather than leaving a number of gaps in the RVOD product, a radar-onlly (RO) retrieval is performed for those profiles where visible optical depth information is not available (as indicated by status flags in 2B-TAU). (Retrieved profiles based on an RO retrieval are indicated by a non-zero value of bit 7 in `RVOD_CWC_status`: "Bad TAU input".) The RVOD product is therefore a composite of RVOD and RO retrieval solutions in which each retrieved profile uses the maximum information available through the CloudSat data system. For users wanting a more homogenous product, the 2B-CWC-RO product (using radar only) continues to be available.

The CWC algorithm creates a composite profile from separate ice and liquid water retrievals. Both of these retrievals assume that the radar profile is due to a single phase of water, that is, that the entire profile consists of either liquid or ice, but not both. The resulting separate liquid and ice profiles are then combined using a simple scheme based on temperature as reported by an ECMWF model. While the combination algorithm results in a mixture of ice and liquid phases over that part of the vertical profile that has the proper temperature range, the user should be aware that the retrieval does not attempt to retrieve mixed-phase cloud properties directly. Improvements are in the planning stages to better handle the retrieval of mixed-phase cloud.

This document describes the algorithms that have been implemented in Release 4 (R04) of the 2B-CWC-RVOD product (algorithm version 5.1). For each radar profile, the algorithms will

- Examine the cloud mask in 2B-GEOPROF to determine which bins in the column contain cloud,

- Examine the 2B-CLDCLASS product to determine if any cloudy bins have an undetermined or invalid cloud type (indicating a problematic profile),

- Examine the 2B-TAU product to determine if the visible optical depth retrieval is bad, missing, or failed to converge,

- Assign *a priori* values to the liquid and ice particle size distribution parameters in each cloudy bin based on climatology, temperature, or other criteria,

- Using the *a priori* values and radar measurements from 2B-GEOPROF (and visible optical depth from 2B-TAU, if available), retrieve liquid and ice particle size distribution parameters for each cloudy bin. Derive effective radius, water content, and related quantities from the retrieved size distributions for both liquid and ice phases, together with associated uncertainties,

- Create a composite profile by using the retrieved ice properties at temperatures colder than $-20°C$, the retrieved liquid properties at temperatures warmer than $0°C$, and a linear combination of the two in intermediate temperatures,

- For each of these estimates, calculate uncertainties and covariance matrices.

2 Algorithm Theoretical Basis—Liquid Water Content

The liquid cloud retrieval algorithm is a modification of the method described in Austin and Stephens (200X). [A previous version of the algorithm is described in Austin and Stephens (2001).] Condensed versions of the descriptions of the forward model and retrieval formulation are given here and in the following section, together with a description of modifications specific to the operational CloudSat algorithm.

2.1 Forward Model and Measurements

The retrieval uses active and passive remote sensing data together with *a priori* data to estimate the parameters of the particle size distribution in each bin containing cloud. Radar measurements provide a vertical profile of cloud backscatter; the measured backscatter value and a cloud mask (indicating the likelihood that a particular radar bin contains cloud) are obtained from 2B-GEOPROF. Retrieved visible optical depth values provide an estimate of the integrated extinction through the cloud column; visible optical depth values and uncertainties are obtained from 2B-TAU.

2.1.1 Physics of the Forward Model

The forward model developed for the retrieval assumes a lognormal size distribution of cloud droplets:

$$N(r) = \frac{N_T}{\sqrt{2\pi}\omega r} \exp\left[\frac{-\ln^2(r/r_g)}{2\omega^2}\right], \tag{1}$$

where N_T is the droplet number density, r is the droplet radius, and r_g, ω, and σ_g are defined by

$$\begin{aligned} \ln r_g &= \overline{\ln r}, \\ \omega &= \ln \sigma_g, \\ \sigma_g^2 &= \overline{(\ln r - \ln r_g)^2}, \end{aligned}$$

where r_g is the geometric mean radius, ω is the distribution width parameter, σ_g is the geometric standard deviation, $\ln$ indicates the natural (base e) logarithm, and the overbar indicates the arithmetic mean. The distribution in (1) is fully specified by three parameters: N_T, ω, and r_g. The liquid water content LWC and the effective radius r_e are defined in terms of moments of the size distribution:

$$\text{LWC} = \int_0^\infty \rho_w N(r) \frac{4}{3}\pi r^3 \, dr, \tag{2}$$

$$r_e = \frac{\int_0^\infty N(r) r^3 \, dr}{\int_0^\infty N(r) r^2 \, dr}, \tag{3}$$

where ρ_w is the density of water.

For clouds having negligible drizzle or precipitation, cloud droplets are sufficiently small to be modeled as Rayleigh scatterers at the CloudSat radar wavelength and sufficiently large that their extinction efficiency approaches 2 for visible wavelengths. These assumptions yield the following definitions of radar reflectivity factor Z and visible extinction coefficient σ_{ext}:

$$Z = 64 \int_0^\infty N(r) r^6 \, dr, \tag{4}$$

$$\sigma_{\text{ext}} = 2 \int_0^\infty N(r)\pi r^2 \, dr. \tag{5}$$

Using (1) for the size distribution in (2) through (5) gives the following equations for the various cloud properties:

$$\text{LWC} = \frac{4\pi}{3} N_T \rho_w r_g^3 \exp\left(\frac{9}{2}\omega^2\right), \tag{6}$$

$$r_e = r_g \exp\left(\frac{5}{2}\omega^2\right), \tag{7}$$

$$Z = 64 N_T r_g^6 \exp(18\omega^2), \tag{8}$$

$$\sigma_{\text{ext}} = 2\pi N_T r_g^2 \exp(2\omega^2). \tag{9}$$

All of these properties are functions of position in the cloud column; we can therefore write $\text{LWC}(z)$, $r_e(z)$, $Z(z)$, and $\sigma_{\text{ext}}(z)$.

The visible optical depth τ is calculated by integrating the visible extinction coefficient through the cloud column:

$$\tau = \int_{z_{\text{base}}}^{z_{\text{top}}} \sigma_{\text{ext}}(z)\, dz, \tag{10}$$

where z_{base} and z_{top} are the cloud base and top, respectively. Equations (6) through (10) express the intrinsic properties of the cloud as functions of the parameters of the assumed drop size distribution; they form the basis of the retrieval. LWC and r_e are the quantities we seek to retrieve, and values of Z are related to our measurements. We may also specify LWP, the columnar liquid water content or liquid water path,

$$\text{LWP} = \int_{z_{\text{base}}}^{z_{\text{top}}} \text{LWC}(z)\, dz. \tag{11}$$

The scattered energy received by the radar from particles at a given range will be attenuated in both directions by cloud particles between that range and the radar receiver. (It will also experience gaseous attenuation, primarily by water vapor; this attenuation is provided as a separate variable in the 2B-GEOPROF product and is therefore not considered here.) The measured reflectivity factor Z' will be reduced from the intrinsic reflectivity factor Z according to the following expression:

$$Z'(z) = Z(z) \exp\left[-2 \int_{\text{path}} \sigma_{\text{abs}}(z')\, dz'\right], \tag{12}$$

where the path integral is over the portion of the cloud between z and the radar. The absorption coefficient at the radar frequency σ_{abs} is given by

$$\sigma_{\text{abs}}(z) = \int_0^\infty N(r, z) C_{\text{abs}}(r)\, dr, \tag{13}$$

where $N(r, z)$ is the particle size distribution at z and C_{abs} is the absorption cross section as a function of particle radius r. (Scattering effects are much smaller than absorption effects at the radar wavelength, so we approximate the attenuation as being purely due to absorption.)

Assuming the cloud droplets are sufficiently small to be modeled as spherical droplets, we may use Mie theory to obtain an expression for C_{abs}:

$$C_{\text{abs}} = \frac{8\pi^2 r^3}{\lambda} \text{Im}\{-K\}, \tag{14}$$

where λ is the radar wavelength and K is given by

$$K = \frac{m^2 - 1}{m^2 + 2}, \tag{15}$$

where m is the complex index of refraction of the droplet material (liquid water) at the radar frequency and ambient temperature. Using the lognormal distribution in (1), the absorption coefficient in (13) becomes

$$\sigma_{\mathrm{abs}} = \frac{8\pi^2 N_T}{\lambda} \mathrm{Im}\{-K\} r_g^3 \exp\left(\frac{9}{2}\omega^2\right),\tag{16}$$

where the z dependence is suppressed for clarity.

Assuming that a lognormal distribution is appropriate, the cloud microphysics are fully described by specification of the three lognormal distribution parameters $N_T(z)$, $\omega(z)$, and $r_g(z)$. Liquid water content and effective radius may then be obtained through (6) and (7). Because the measured data are limited to a single radar reflectivity factor Z' for each radar resolution bin and a visible optical depth τ for the entire cloud column, we rely on *a priori* data to constrain the retrieval where the measurements cannot, allowing the retrieved solution to be consistent with the measurements without imposing fixed values of (e.g.) particle number concentration through the cloud column. The optimal estimation technique employed in this retrieval is described in section 2.2.

2.1.2 Departures From the Lognormal Distribution

The retrieval assumes a lognormal distribution of liquid cloud droplets, as given in (1). Departures from this distribution will degrade the accuracy of the retrieval. One source of departure from this analytic distribution is the presence of drizzle or rain within the cloud. Detection criteria for the presence of drizzle or rain are under development. (The current procedure identifies drizzle or precipitation for any case where $Z' \geq -15$ dBZ.) Drizzle/precipitation is indicated in the output by setting a flag in the status variable, but the algorithm is still run as normal, producing output values (unless the solution diverges). The flag serves as an indicator that the solution is likely unreliable due to a violation of the lognormal distribution assumption. In practice, the presence of any significant precipitation often causes the retrieval to fail to converge, resulting in an error condition. Retrieval of cloud properties in the presence of precipitation is a difficult problem due to the sensitivity of the radar to precipitation-sized particles. Better handling of this case is a high priority for future development of this product.

2.1.3 Mixed phase and multi-layered clouds

The liquid water content retrieval algorithm assumes that the entire cloud column is composed of liquid water droplets. (The retrievals used in 2B-CWC-RVOD consider only one phase of water at a time.) Because we have no independent means of determining the particle phase in a given radar bin, a simple partition scheme is employed to create a composite ice/liquid profile, discarding the retrieved liquid properties in portions of the profile deemed to consist purely of ice. The partition scheme is described in section 4.

2.2 Retrieval Algorithm

The retrieval uses an approach described by Rodgers (1976, 1990, 2000) and Marks and Rodgers (1993), where a vector of measured quantities $\mathbf{y}$ (here, radar reflectivities and visible optical depth) is related to a state vector of unknowns $\mathbf{x}$ (geometric mean radii, number density, and distribution width parameter) by the forward model $\mathbf{F}$:

$$\mathbf{y} = \mathbf{F}(\mathbf{x}) + \epsilon_{\mathbf{y}},\tag{17}$$

where $\epsilon_{\mathbf{y}}$ represents measurement errors. Rodgers (1976) described an optimal-estimation technique in which *a priori* profiles are used as virtual measurements, serving as a constraint on the retrieval. An *a priori* profile $\mathbf{x_a}$ is specified based on likely or statistical values of the state vector elements, together

with an *a priori* covariance matrix $\mathbf{S_a}$ representing the variability or uncertainty of this profile and the covariance between various profile elements.

The retrieval algorithm obtains the optimal solution by minimizing a cost function Φ that represents a weighted sum of the measurement vector-forward model difference and the state vector-*a priori* difference:

$$\Phi = (\hat{\mathbf{x}} - \mathbf{x_a})^{\mathrm{T}}\mathbf{S_a}^{-1}(\hat{\mathbf{x}} - \mathbf{x_a}) + [\mathbf{y} - \mathbf{F}(\hat{\mathbf{x}})]^{\mathrm{T}}\mathbf{S_y}^{-1}[\mathbf{y} - \mathbf{F}(\hat{\mathbf{x}})]. \tag{18}$$

The solution is obtained by iteration using successive estimates of the $\mathbf{x}$ vector and the $\mathbf{K}$ matrix ($\mathbf{K} = \partial\mathbf{F}/\partial\mathbf{x}$). These quantities are also used to provide information on convergence, the quality of the solution, and the amounts and sources of retrieval uncertainty. The various input and output quantities are described here; see Austin and Stephens (200X) for a more detailed description.

2.2.1 State and Measurement Vectors

The state vector $\mathbf{x}$ is the vector of unknown cloud parameters to be retrieved. For a cloud reflectivity profile consisting of p cloudy bins, the state vector will have $n = 3p$ elements:

$$\mathbf{x} = \begin{bmatrix} r_g(z_1) \\ \vdots \\ r_g(z_p) \\ N_T(z_1) \\ \vdots \\ N_T(z_p) \\ \omega(z_1) \\ \vdots \\ \omega(z_p) \end{bmatrix}, \tag{19}$$

where $r_g(z_i)$, $N_T(z_i)$, and $\omega(z_i)$ are the geometric mean radius, droplet number concentration, and distribution width parameter for height z_i (we shall often write these as r_{g_i}, etc.). Here z_1 is the height of the radar resolution bin at cloud base; z_p is at the top of the cloud profile. Units are selected to keep the numerical values within similar orders of magnitude: μm for r_g and cm^{-3} for N_T (ω is dimensionless).

The RVOD measurement vector $\mathbf{y}$ is composed of $m = p + 1$ elements for a cloud profile of p cloudy bins:

$$\mathbf{y} = \begin{bmatrix} Z'_{\mathrm{dB}}(z_1) \\ \vdots \\ Z'_{\mathrm{dB}}(z_p) \\ \tau \end{bmatrix}, \tag{20}$$

where $Z'_{\mathrm{dB}}(z_i)$ is the measured radar reflectivity factor for height z_i (often written as Z'_{dB_i}) and τ is the input visible optical depth. Reflectivity factor Z is specified in units of mm^6 m^{-3}. To reduce the large dynamic range of the reflectivity variable and to make the model more linear, Z has been converted to a logarithmic variable Z_{dB} by the transform $Z_{\mathrm{dB}} = 10\log Z$, where Z_{dB} has units of dBZ and $\log$ indicates the base 10 logarithm.

The measurement error covariance matrix $\mathbf{S_\epsilon}$ gives a measure of the uncertainties in the measurement vector and of correlations between the errors of the individual elements. In the present retrieval,

it is assumed that the elements of $\mathbf{y}$ have independent errors given as follows:

$$
\mathbf{S}_\epsilon = \begin{bmatrix}
\sigma^2_{Z'_{\mathrm{dB}_1}} & 0 & \cdots & 0 & 0 \\
0 & \ddots & 0 & \vdots & \vdots \\
\vdots & 0 & \ddots & 0 & 0 \\
0 & \cdots & 0 & \sigma^2_{Z'_{\mathrm{dB}_p}} & 0 \\
0 & \cdots & 0 & 0 & \sigma^2_\tau
\end{bmatrix},
\tag{21}
$$

where $\sigma_{Z'_{\mathrm{dB}_i}}$ is the standard deviation of the measured radar reflectivity factor in dBZ (i.e., the uncertainty in the measured radar reflectivity values from whatever source [noise, calibration error, etc.], usually a fixed number for a given radar), and σ_τ is the standard deviation of the retrieved visible optical depth obtained from 2B-TAU for each individual profile.

2.2.2 Forward Model and Parameters

The forward model $\mathbf{F}(\mathbf{x})$ relates the state vector $\mathbf{x}$ to the measurement vector $\mathbf{y}$. $\mathbf{F}$ therefore has the same dimension as $\mathbf{y}$:

$$
\mathbf{F}(\mathbf{x}) = \begin{bmatrix}
Z'_{\mathrm{dB}_{\mathrm{FM}}}(z_1) \\
\vdots \\
Z'_{\mathrm{dB}_{\mathrm{FM}}}(z_p) \\
\tau_{\mathrm{FM}}
\end{bmatrix},
\tag{22}
$$

where the individual elements are given by the following expressions:

$$
\begin{aligned}
Z'_{\mathrm{dB}_{\mathrm{FM}}}(z_i) = \\
10 \log \Bigg\{ 64 N_{T_i} r^6_{g_i} \exp(18\omega_i^2) \\
\times \exp \Bigg[\frac{-16\pi^2 N_{T_i}}{\lambda} \mathrm{Im}\{-K\} \\
\times \exp \left(\frac{9}{2} \omega_i^2 \right) \Delta z \sum_{j=i+1}^{p} r^3_{g_j} \Bigg] \Bigg\}, \qquad i = 1, \ldots, p-1
\end{aligned}
\tag{23}
$$

$$
Z'_{\mathrm{dB}_{\mathrm{FM}}}(z_i) = 10 \log \left[64 N_{T_i} r^6_{g_i} \exp(18\omega_i^2) \right], \quad i = p
\tag{24}
$$

$$
\tau_{\mathrm{FM}} = \sum_{i=1}^{p} \left[2\pi N_{T_i} r^2_{g_i} \exp(2\omega_i^2) \Delta z \right]
\tag{25}
$$

(The symbol Δz represents the thickness of a radar range bin.) The subscript FM is a reminder that these quantities are calculated from elements of $\mathbf{x}$ according to the forward model equations (23), (24), and (25), as opposed to the elements of the $\mathbf{y}$ vector, which are measured quantities. The form of (23) and (24) assumes that the radar is above the cloud looking down; again, z_1 is the lowest bin in the cloud and z_p is the highest.

2.2.3 A Priori Data and Covariance

A priori data for the retrieval are selected based on collections of microphysical measurements of related cloud types. Reference values for each of these categories are obtained from a database of cloud microphysical parameters (e.g., Miles et al. 2000); in the current version, a common set of

values are used for all cloud types. The selected values for each radar profile are included in the product output. The *a priori* vector $\mathbf{x_a}$ is specified as follows:

$$\mathbf{x_a} = \begin{bmatrix} r_{ga}(z_1) \\ \vdots \\ r_{ga}(z_p) \\ N_{Ta}(z_1) \\ \vdots \\ N_{Ta}(z_p) \\ \omega_a(z_1) \\ \vdots \\ \omega_a(z_p) \end{bmatrix}. \tag{26}$$

We also specify an *a priori* error covariance matrix $\mathbf{S_a}$:

$$\mathbf{S_a} = \begin{bmatrix}
\sigma^2_{r_{ga_1}} & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \ddots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & 0 & \sigma^2_{r_{ga_p}} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & \sigma^2_{N_{Ta_1}} & 0 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \cdots & \vdots & 0 & \ddots & 0 & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & \cdots & 0 & \sigma^2_{N_{Ta_p}} & 0 & \vdots & \vdots \\
0 & \cdots & 0 & \cdots & \cdots & 0 & \sigma^2_{\omega_{a_1}} & 0 & \vdots \\
\vdots & \cdots & \vdots & \cdots & \cdots & \cdots & 0 & \ddots & 0 \\
0 & \cdots & 0 & \cdots & \cdots & \cdots & \cdots & 0 & \sigma^2_{\omega_{a_p}}
\end{bmatrix}. \tag{27}$$

Adjustment of the *a priori* parameters $\mathbf{x_a}$ and uncertainties $\mathbf{S_a}$ in future versions may allow customization of the retrieval for different cloud types, generation regimes (e.g., continental or maritime), and geographic areas (tropical, midlatitude, etc.).

2.2.4 Convergence and Quality Control

The state vector $\hat{\mathbf{x}}$ is obtained by iteration. The *a priori* values $\mathbf{x_a}$ are used as the initial value of $\hat{\mathbf{x}}$. Convergence of the solution is determined using a test with the following form:

$$\Delta\hat{\mathbf{x}}^{\mathrm{T}}\mathbf{S_x}^{-1}\Delta\hat{\mathbf{x}} \ll n, \tag{28}$$

where n is the dimension of the $\hat{\mathbf{x}}$ vector, i.e. the number of cloudy radar bins times three. The error covariance matrix $\mathbf{S_x}$ of the retrieved state vector $\hat{\mathbf{x}}$ is given by

$$\mathbf{S_x} = (\mathbf{S_a}^{-1} + \mathbf{K}^{\mathrm{T}}\mathbf{S_y}^{-1}\mathbf{K})^{-1}. \tag{29}$$

Elements of the $\mathbf{S_x}$ matrix give the covariance between elements of the retrieved state vector $\hat{\mathbf{x}}$; diagonal elements of $\mathbf{S_x}$ are variances in the elements of $\hat{\mathbf{x}}$ and give a measure of the uncertainty in the retrieval. For this retrieval, we specify the criterion for "much less than" in (28) such that

$$\Delta\hat{\mathbf{x}}^{\mathrm{T}}\mathbf{S_x}^{-1}\Delta\hat{\mathbf{x}} < 0.01n. \tag{30}$$

After the iteration converges, we seek a test that shows the goodness of fit of the retrieved values to the measurements. Using the hypothesis that the fit to the measurements (including the *a priori* virtual measurements) is consistent with the measurement uncertainties (including the *a priori* uncertainties), Marks and Rodgers (1993) used the following χ^2:

$$\chi^2 = [\mathbf{y} - \mathbf{F}(\hat{\mathbf{x}})]^T \mathbf{S_y}^{-1} [\mathbf{y} - \mathbf{F}(\hat{\mathbf{x}})] + (\mathbf{x_a} - \hat{\mathbf{x}})^T \mathbf{S_a}^{-1} (\mathbf{x_a} - \hat{\mathbf{x}}). \tag{31}$$

This quantity should follow a χ^2 distribution with m degrees of freedom (n parameters fitted to $m + n$ measurements, where n and m are the dimensions of the $\hat{\mathbf{x}}$ and $\mathbf{y}$ vectors, respectively). Marks and Rodgers (1993) noted that a typical value of χ^2 for a "moderately good retrieval" is m. The quantity χ^2/m is included as an output field in 2B-CWC-RVOD.

As currently implemented, the retrieval rejects profiles where any element of the $\hat{\mathbf{x}}$ vector becomes negative during any iteration. (This is infrequent.) Profiles are also rejected if the measured reflectivity factor exceeds a specified maximum level (i.e., it is unphysical) or if the radar information is unavailable.

2.2.5 Radar Uncertainty Changes from 2B-CWC-RO

Release 4 of 2B-CWC-RVOD introduces a change in the representation of radar uncertainty. The original scheme (used in 2B-CWC-RO) attempted to model the uncertainty of CloudSat's Cloud Profiling Radar according to an early estimate of the ratio of the standard deviation to the mean signal as a function of true reflectivity. The resulting approximate uncertainty σ_Z in the measured reflectivity factor was therefore large for weak signals but very small for strong signals, as shown by the black curve in Figure 1.

The new uncertainty scheme (shown in red in Figure 1) differs from the former scheme in two ways. First, the radar is more sensitive than originally estimated, so the uncertainty σ_Z has lower values at the weak signal end. Second, we now attempt to include the uncertainty in the absolute calibration of the radar by setting the minimum value of σ_Z to 2 dBZ for all signal strengths. Thus the new scheme has increased uncertainty for strong signals but reduced uncertainty for weak signals. While the accuracy of the new uncertainty scheme is still to be determined, one definite benefit is a marked decrease in the number of profiles where the retrieval fails, particularly in the liquid case. The looser constraint on fitting the measured data, particularly for strong radar signals, allows the retrieval to converge on a solution much more often than was possible with the former method. If no major problems are found with this new scheme in 2B-CWC-RVOD, then it will be adopted into the 2B-CWC-RO product in its next version.
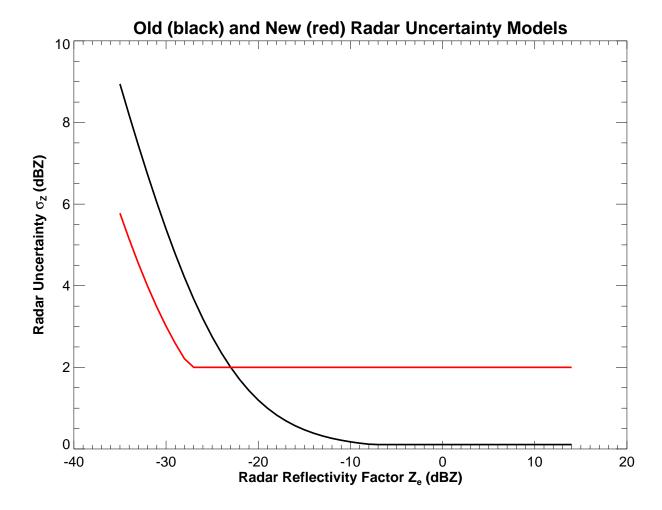
Figure 1: A comparison of the old (black) and new (red) models of radar uncertainty as a function of radar reflectivity. The old model is used in 2B-CWC-RO; 2B-CWC-RVOD uses the new model.

3 Algorithm Theoretical Basis—Ice Water Content

The retrieval algorithm is a modification of the RO retrieval described in Austin et al. (2008, in press); an earlier version is described by Benedetti et al. (2003). Condensed versions of the descriptions of the forward model and retrieval formulation are given here and in the following section, together with a description of modifications specific to the operational CloudSat algorithm.

3.1 Forward Model and Measurements

Like the liquid cloud retrieval, this retrieval uses active and passive remote sensing data together with *a priori* data to estimate the parameters of the particle size distribution in each bin containing cloud. Radar measurements provide a vertical profile of cloud backscatter; the measured backscatter value and a cloud mask (indicating the likelihood that a particular radar bin contains cloud) are obtained from 2B-GEOPROF. Retrieved visible optical depth values provide an estimate of the integrated extinction through the cloud column; visible optical depth values and uncertainties are obtained from 2B-TAU.

3.1.1 Physics of the Forward Model

The forward model developed for the retrieval assumes a lognormal size distribution of ice crystals:

$$N(D) = \frac{N_T}{\sqrt{2\pi}\omega D} \exp\left[\frac{-\ln^2(D/D_g)}{2\omega^2}\right], \tag{32}$$

where N_T is the ice particle number concentration, D is the diameter of an equivalent mass ice sphere, D_g is the geometric mean diameter, and ω is the width parameter. The distribution in (32) is fully specified by three parameters: N_T, D_g, and ω. The ice water content (IWC) and the effective radius r_e are defined in terms of moments of the size distribution:

$$\text{IWC} = \int_0^\infty \rho_i \frac{\pi}{6} N(D) D^3 dD \tag{33}$$

$$r_e = \frac{1}{2} \frac{\int_0^\infty N(D) D^3 \, dD}{\int_0^\infty N(D) D^2 \, dD}, \tag{34}$$

where ρ_i is the density of ice.

For thin ice clouds, the cloud ice particles are sufficiently small to be modeled as Rayleigh scatterers at the CloudSat radar wavelength and sufficiently large that their extinction efficiency approaches 2 for visible wavelengths. These assumptions yield the following definitions of radar reflectivity factor Z and visible extinction coefficient σ_{ext}:

$$Z_{\text{Ray}} = \int_0^\infty N(D) D^6 dD \tag{35}$$

$$\sigma_{\text{ext}} = 2\int_0^\infty N(D)\frac{\pi}{4}D^2 dD \tag{36}$$

Using (32) for the size distribution in (33) through (36) gives the following equations for the various cloud properties:

$$\text{IWC} = \rho_i \frac{\pi}{6} N_T D_g^3 \exp\left(\frac{9}{2}\omega^2\right) 10^{-3} \tag{37}$$

$$r_e = \frac{1}{2} D_g \exp\left(\frac{5}{2}\omega^2\right) 10^3 \tag{38}$$

$$\sigma_{\text{ext}} = \frac{\pi}{2} N_T D_g^2 \exp(2\omega^2) 10^{-3} \tag{39}$$

$$Z_{\text{Ray}} = N_T D_g^6 \exp(18\omega^2), \tag{40}$$

All of these properties are functions of position in the cloud column; we can therefore write $\mathrm{IWC}(z)$, $r_e(z)$, $\sigma_{\mathrm{ext}}(z)$, and $Z_{\mathrm{Ray}}(z)$.

The visible optical depth τ is calculated by integrating the visible extinction coefficient through the cloud column:

$$\tau = \int_{z_{\mathrm{base}}}^{z_{\mathrm{top}}} \sigma_{\mathrm{ext}}(z)\, dz, \tag{41}$$

where z_{base} and z_{top} are the cloud base and top, respectively. Equations (37) through (41) express the intrinsic properties of the cloud as functions of the parameters of the assumed particle size distribution; they form the basis of the retrieval. The parameters IWC and r_e are the quantities we seek to retrieve, and values of Z are related to our measurements. We may also specify the columnar ice water content or ice water path (IWP),

$$\mathrm{IWP} = \int_{z_{\mathrm{base}}}^{z_{\mathrm{top}}} \mathrm{IWC}(z)\, dz. \tag{42}$$

The three parameters $N_T(z)$, $D_g(z)$, and $\omega(z)$ fully define the size distribution. Ice water content and effective radius may then be obtained through (37) and (38). Because the measured data are limited to a single radar reflectivity factor Z' for each radar resolution bin and a visible optical depth τ for the entire cloud column, we rely on *a priori* data to constrain the retrieval where the measurements cannot, allowing the retrieved solution to be consistent with the measurements without imposing fixed values of (e.g.) particle number concentration through the cloud column. The optimal estimation technique employed in this retrieval is described in section 3.2.

3.1.2 Algorithm refinements: correction for Lorenz-Mie effects

At frequencies of radars commonly used for cirrus cloud detection (35 or 94 GHz), the size parameter (the ratio between the diameter of the particle D and the radar wavelength λ) remains smaller than unity for crystal sizes up to 100 μm (and even larger for 35 GHz). Therefore, the Rayleigh approximation is almost always satisfied at these frequencies. However, because radar reflectivity in the Rayleigh regime is a function of the *sixth* power of the particle diameter, the error introduced by use of the Rayleigh approximation on the large crystals that violate the Rayleigh criterion may be significant, even if these coarser particles are few in number. To quantify this error, we performed Lorenz-Mie calculations and parameterized the ratio of the (exact) Lorenz-Mie radar reflectivity to the (approximate) Rayleigh reflectivity in terms of the size distribution parameters. Starting from the most general form of the radar equation, we have

$$\frac{P_r}{P_t} = \frac{\tilde{C}\lambda^2}{R^2} \int n(D) C_b(D) dD, \tag{43}$$

where $\tilde{C}$ is the generalized radar constant and $C_b(D)$ is the *backscattering* coefficient. In the Rayleigh limit, $C_b(D)$ takes the form

$$C_b(D) = \frac{\pi^5}{\lambda^4} |K|^2 D^6, \tag{44}$$

where K is given by

$$K = \frac{m^2 - 1}{m^2 + 2}, \tag{45}$$

where m is the complex index of refraction of the particle material (water ice) at the radar frequency and ambient temperature. The dependence on the sixth power of the diameter in the radar reflectivity definition derives from (44).

The Lorenz-Mie theory provides an exact expression for C_b for homogeneous spheres that can be used instead of (44) to define an equivalent "Mie" radar reflectivity, Z_{Mie}. We computed Z_{Mie} using

a code provided by Bohren and Huffman (1983) and plotted the ratio of Z_{Mie} and Z_{Ray} as a function of the distribution parameters D_g and ω. At small particle sizes, the ratio is unity, indicating that the Rayleigh approximation is valid. For larger sizes, the two quantities begin to diverge, but the shape of the ratio function is well fitted by the following combination of Gaussian functions:

$$f_{\mathrm{Mie}}(D_g, \omega) = \frac{Z_{\mathrm{Mie}}}{Z_{\mathrm{Ray}}} = A_0 \exp\left[-\frac{1}{2}\left(\frac{D_g}{A_1} \right)^2 \right] + A_2 \tag{46}$$

where

$$A_0 = a_{01} + a_{02} \exp\left[-\frac{1}{2}\left(\frac{\omega - 1}{a_{03}} \right)^2 \right] \tag{47}$$

$$A_1 = a_{11}(\omega - 1)^2 + a_{12} \tag{48}$$

$$A_2 = a_{21}(\omega - 1)^2 + a_{22} \tag{49}$$

where the coefficients a_{01}, a_{02}, a_{03}, a_{11}, a_{12}, a_{21}, and a_{22} have values of 0.99, -0.965, 0.25, 0.9688, 0.02, 0.0625, and 0.000001, respectively. The expression f_{Mie} derived to account for the Lorenz-Mie effects has analytical properties and is differentiable, as is the radar forward model of (40). The new forward model can be written as

$$Z = Z_{\mathrm{Ray}} f_{\mathrm{Mie}}(D_g, \omega). \tag{50}$$

3.1.3 Further possible algorithm refinements: correction for density effects

Radar reflectivity is conventionally defined with respect to water Z_e, even when the radar target is known to be a volume of ice particles. To transform the ice quantities into equivalent radar reflectivity with respect to water, a constant correction factor defined as the ratio of K_{ice} and K is introduced, where these constants are proportional to the refractive index. In so doing, an implicit assumption is also made that the density of ice crystals is constant. The refractive index from porous ice particles such as large snow flakes/aggregates is generally considered to be some mixture of ice and air and is thus reduced in value from that of solid ice. Future versions of the retrieval will attempt to treat the effects of porosity by making K_{ice} a function of density. [Matrosov (1999) discusses this problem in great detail.] In the current version of the algorithm, no density correction is implemented. The equivalent reflectivity factor is thus written

$$Z_e = Z_{\mathrm{Ray}} f_{\mathrm{Mie}}(D_g, \omega) \tilde{K} \tag{51}$$

where $\tilde{K} = 0.232$ is a fixed correction factor (Stephens 1994).

3.1.4 Departures from the Lognormal Distribution

The retrieval assumes a lognormal distribution of ice particles, as given in (32). Departures from this distribution will degrade the accuracy of the retrieval. One source of departure from this analytic distribution is the presence of large particles within the cloud that may introduce a bimodality in the particle spectra (e.g., in thick anvil cirrus). Reflectivities greater than -15 dBZ are indicated in the output by setting a flag; but the algorithm is still run as normal, producing output values (unless the solution diverges). The flag serves as an indicator that the solution is likely unreliable due to a violation of the lognormal distribution assumption.

3.1.5 Mixed phase and multi-layered clouds

The ice water content retrieval algorithm assumes that the entire cloud column is composed of ice particles, except for bins having ECMWF-AUX temperatures warmer than $1°C$, which are omitted from the retrieval. (The retrievals used in 2B-CWC-RVOD consider only one phase of water at a time.) Because we have no independent means of determining the particle phase in a given radar bin, a simple partition scheme is employed to create a composite ice/liquid profile, discarding the retrieved ice properties in portions of the profile deemed to consist purely of liquid. The partition scheme is described in section 4.

3.1.6 Radar Uncertainty Changes from 2B-CWC-RO

The changed representation of radar uncertainty described in section 2.2.5 also affect the ice cloud retrieval. As in the liquid case, the new uncertainty model will be incorporated into the next version of 2B-CWC-RO if no major problems are found in its application to 2B-CWC-RVOD.

3.2 Retrieval Algorithm

The ice retrieval uses the same optimal estimation framework used by the liquid retrieval as described in section 2.2. The various input and output quantities are described here; see Austin et al. (2008, in press) and Benedetti et al. (2003) for a more detailed description.

3.2.1 State and Measurement Vectors

The state vector $\mathbf{x}$ is the vector of unknown cloud parameters to be retrieved. For a cloud reflectivity profile consisting of p cloudy bins, the state vector will have $n = 3p$ elements:

$$\mathbf{x} = \begin{bmatrix} \log_{10} D_g(z_1) \\ \vdots \\ \log_{10} D_g(z_p) \\ \log_{10} N_T(z_1) \\ \vdots \\ \log_{10} N_T(z_p) \\ \omega(z_1) \\ \vdots \\ \omega(z_p) \end{bmatrix}, \tag{52}$$

where $D_g(z_i)$, $N_T(z_i)$, and $\omega(z_i)$ are the geometric mean diameter, number concentration, and distribution width parameter for height z_i (we shall often write these as D_{g_i}, etc.). Here z_1 is the height of the radar resolution bin at cloud base; z_p is at the top of the cloud profile. The units of D_g are mm and the units of N_T are m^{-3}; ω is dimensionless.

The measurement vector $\mathbf{y}$ is identical to that used in the liquid retrieval, composed of $m = p + 1$ elements for a cloud profile of p cloudy bins:

$$\mathbf{y} = \begin{bmatrix} Z'_{\text{dB}}(z_1) \\ \vdots \\ Z'_{\text{dB}}(z_p) \\ \tau \end{bmatrix}, \tag{53}$$

where $Z'_{\mathrm{dB}}(z_i)$ is the measured radar reflectivity for height z_i (often written as Z'_{dB_i}). Reflectivity is specified in units of mm^6 m^{-3}. To reduce the large dynamic range of the reflectivity variable and to make the model more linear, Z has been converted to a logarithmic variable Z_{dB} by the transform $Z_{\mathrm{dB}} = 10 \log Z$, where Z_{dB} has units of dBZ and $\log$ indicates the base 10 logarithm. Because ice particle attenuation is small (compared to attenuation by liquid particles), ice particle attenuation effects are neglected in this version of 2B-CWC-RVOD.

The measurement error covariance matrix $\mathbf{S}_\epsilon$ gives a measure of the uncertainties in the measurement vector and of correlations between the errors of the individual elements. It is identical to that used in the liquid retrieval (21).

3.2.2 Forward Model and Parameters

The forward model $\mathbf{F}(\mathbf{x})$ relates the state vector $\mathbf{x}$ to the measurement vector $\mathbf{y}$. $\mathbf{F}$ therefore has the same dimension as $\mathbf{y}$:

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} Z'_{\mathrm{dB}_{\mathrm{FM}}}(z_1) \\ \vdots \\ Z'_{\mathrm{dB}_{\mathrm{FM}}}(z_p) \\ \tau \end{bmatrix}, \tag{54}$$

where the individual elements are given by the following expression:

$$Z'_{\mathrm{dB}_{\mathrm{FM}}}(z_i) = 10 \log[N_{T_i} D_{g_i}^6 \exp(18\omega_i^2) f_{\mathrm{Mie}}(D_{g_i}, \omega)\tilde{K}], \quad i = 1, ..., p \tag{55}$$

$$\tau_{\mathrm{FM}} = \sum_{i=1}^{p} \frac{\pi}{2} N_{T_i} D_{g_i}^2 \exp(2\omega_i^2)\Delta z \tag{56}$$

where the symbol Δz represents the spacing between radar range bins. The subscript FM is a reminder that these quantities are calculated from elements of $\mathbf{x}$ according to the forward model equations (55) and (56), as opposed to the elements of the $\mathbf{y}$ vector, which are measured quantities.

3.2.3 A Priori Data and Covariance

A priori data values for the ice retrieval are selected in two ways. Values of D_{g_i} and ω_i are determined using temperature-based parameterizations constructed from collections of ice particle size distribution measurements from aircraft flights during recent field campaigns. The values therefore vary through the cloud profile according to the temperature indicated by the CloudSat ECMWF-AUX data product. A different procedure was used to set the *a priori* N_T value. Rather than using a temperature-based value for this parameter, it was recognized that reflectivity values measured by CloudSat are sometimes very different from those predicted by the *a priori* database, even after taking temperature into account. In these cases, the number concentration parameter seemed the logical parameter to account for most of the difference. It was therefore deemed necessary to find a way to obtain a value of N_T that would be "closer" in state space to the set of values consistent with a given measurement. This was accomplished by combining (33), (40), and (51) and solving for N_T. The value of IWC was determined independently from Z_e using a Z-IWC relation from Liu and Illingworth (2000). Values of N_T were obtained for each cloudy bin and then averaged through the profile to obtain a single profile value of N_{Ta}, which was then used in the retrieval process. Uncertainties in all three parameters were then calculated using the values obtained from the aircraft measurement database.

The *a priori* vector $\mathbf{x_a}$ for the ice retrieval is specified as follows:

$$\mathbf{x_a} = \begin{bmatrix} \log_{10} D_{ga}(z_1) \\ \vdots \\ \log_{10} D_{ga}(z_p) \\ \log_{10} N_{Ta}(z_1) \\ \vdots \\ \log_{10} N_{Ta}(z_p) \\ \omega_a(z_1) \\ \vdots \\ \omega_a(z_p) \end{bmatrix}. \tag{57}$$

We also specify an *a priori* error covariance matrix $\mathbf{S_a}$:

$$\mathbf{S_a} = \begin{bmatrix} \sigma^2_{\log_{10} D_{ga_1}} & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & 0 & \sigma^2_{\log_{10} D_{ga_p}} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \sigma^2_{\log_{10} N_{Ta_1}} & 0 & \vdots & \vdots & \vdots & \vdots \\ \vdots & \cdots & \vdots & 0 & \ddots & 0 & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \sigma^2_{\log_{10} N_{Ta_p}} & 0 & \vdots & \vdots \\ 0 & \cdots & 0 & \cdots & \cdots & 0 & \sigma^2_{\omega_{a_1}} & 0 & \vdots \\ \vdots & \cdots & \vdots & \cdots & \cdots & \cdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \cdots & \cdots & \cdots & \cdots & 0 & \sigma^2_{\omega_{a_p}} \end{bmatrix}. \tag{58}$$

3.2.4 Convergence and Quality Control

Convergence criteria for the ice retrieval are identical to those used for the liquid retrieval (see section 2.2.4). The goodness-of-fit statistic is likewise identical.
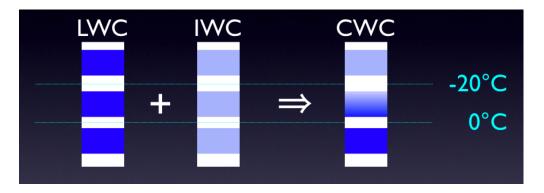
Figure 2: The CWC composite profile is built by combining the retrieved ice and liquid water profiles, according to temperature.

4   Algorithm Theoretical Basis—Cloud Water Content

The original list of CloudSat standard data products included separate products for liquid water content and ice water content. Because there was no independent means of determining the cloud phase in any given radar resolution bin, the plan was to run the liquid and ice retrievals separately on the entire radar profile, resulting in a set of liquid microphysical parameters for each cloudy bin and a corresponding set of ice microphysical parameters for each cloudy bin. The user would then select which answer would be more appropriate or combine the two in some way. No attempt would be made to partition the measured reflectivity between the liquid and ice phases—each solution would assume the entire radar signal was due to a single phase of water.

As the retrievals were further developed and the time approached for the first post-launch data releases, it became clear that this approach would be overly confusing and would likely result in "double-counting" of the cloud water content: users interpreting each cloudy bin as containing *both* liquid and ice water content. To avoid this confusion, a new combined cloud water content product and algorithm were developed. In the new algorithm, the liquid and ice retrievals are run separately on the entire radar profile (as before), but the two resultant profiles are then combined into a composite profile using a simple scheme based on temperature. In this scheme, the portion of the profile colder than $-20°$C is deemed pure ice, so the ice retrieval solution applies there. Similarly, the portion of the profile warmer than $0°$C is considered pure liquid, so the liquid solution applies there. In between these temperatures, the ice and liquid solutions are scaled linearly with temperature (by adjusting the ice and liquid particle number concentrations) to obtain a profile that smoothly transitions from all ice at $-20°$C to all liquid at $0°$C while matching the radar measurements over the whole range.

This scheme gives a very basic partition of the radar measurements into ice and liquid phases. (More sophisticated retrievals for heterogeneous cloud columns are planned for future versions of this product.) The product also contains a 16-bit status variable; individual bits in this variable indicate error conditions in the ice and liquid retrievals and other associated conditions such as large values of the fit parameters or possible precipitation. It is important to note that the partition algorithm is applied separately to the ice and liquid phases regardless of whether both retrievals were successful. For example, if the liquid retrieval fails to converge, the ice water content will still be scaled such that it goes to zero as the temperature increases to $0°$C—there is no attempt to map all the reflectivity to the ice phase to compensate for the failure of the liquid retrieval.

The 2B-CWC-RVOD data product files contain both the composite profiles (which most users will want to use) and the single-phase retrieval profiles (which may be of interest to some investigators). The output data from the liquid cloud retrieval (which interprets all cloud as liquid from the surface to

the stratosphere) is found in fields with names starting with `LO_RVOD_` (for "liquid-only" and "radar-visible optical depth"). Corresponding outputs from the ice cloud retrieval have names starting with `IO_RVOD_`. The fields representing the combination of these into composite profiles have names beginning with `RVOD_liq_` and `RVOD_ice_`. Consult the 2B-CWC-RVOD Interface Control Document for detailed descriptions of the fields contained in the 2B-CWC-RVOD HDF files.

5 Algorithm Inputs

5.1 CloudSat

5.1.1 CloudSat 2B-GEOPROF Data

The CloudSat 2B-GEOPROF product is the principal input for 2B-CWC-RVOD. The retrieval uses the radar reflectivity, the cloud mask, and the gaseous attenuation values from this product. For cases where this input is missing, 2B-CWC-RVOD will have no output.

5.1.2 CloudSat 2B-CLDCLASS Data

Various bits in cloud scenario field in the 2B-CLDCLASS product are used to detect cloud type and to screen problematic profiles. Future versions of the retrievals may use the indicated cloud type, surface type, and other flags to refine the retrieval algorithm, for example by selecting different *a priori* values according to cloud type.

5.1.3 CloudSat 2B-TAU Data

The CloudSat 2B-TAU product provides estimates of the visible optical depth for each cloud column measured by CloudSat, based on MODIS radiances. The 2B-CWC-RVOD retrievals use the retrieved visible optical depth and uncertainty, together with the 2B-TAU status flags indicating the success or failure of the optical depth retrieval. For cases where the optical depth retrieval is not available, 2B-CWC-RVOD uses a radar-only retrieval instead.

5.2 Ancillary (Non-CloudSat)

5.2.1 CloudSat ECMWF-AUX Data

The retrieval uses temperature information from the CloudSat ECMWF-AUX product, which takes model output from ECMWF and interpolates the variables to the CloudSat data grid. Temperature information is used to assign *a priori* values in the ice cloud retrieval and also to guide the combination of the ice and liquid information into composite profiles.

6   Algorithm Summary

The algorithm is implemented in Fortran 90. The following is a pseudocode description of the algorithm implementation:

start 2B-LWC-RVOD

get orbit of 2B-GEOPROF data (CPR cloud mask, radar reflectivity)

get orbit of 2B-CLDCLASS data (cloud scenario)

get orbit of 2B-TAU data (visible optical depth)

for-each 2B-GEOPROF vertical profile

> **convert** bit flags to integer values
>
> **determine** if LWC retrieval will be run (known & valid cloud scenario, cloud present, Z physical— run RVOD if optical depth available, RO otherwise)
>
> **if** running LWC retrieval
>
> > **determine** size of state vector
> > **assign** *a priori* r_g, N_T, and ω values and uncertainties
> > **set** $\mathbf{y}$ vector (2B-GEOPROF) using condensed profile retaining cloudy bins only
> > **set** $\mathbf{S_a}$, $\mathbf{S_a^{-1}}$, $\mathbf{S_\epsilon}$ matrices
> > **repeat**
> >
> > > **calculate** $\mathbf{K}$, $\mathbf{S_y}$, $\mathbf{D_y}$ matrices
> > > **calculate** $\mathbf{F}$ (forward-model) vector
> > > **calculate** $\mathbf{S_y^{-1}}$, $\mathbf{S_x^{-1}}$, $\mathbf{S_x}$ matrices
> > > **calculate** new state vector $\hat{\mathbf{x}}$
> > > **if** $\hat{\mathbf{x}}$ goes negative, reject
> > > **calculate** $\mathbf{\Delta\hat{x}}$, convergence test
> > > **if** more than 15 iterations, reject
> >
> > **end-repeat** until convergetest $< 0.01n$
> > **calculate** r_e, LWC, LWP
> > **calculate** χ^2 and $\mathbf{A}$
> > **calculate** retrieval uncertainties
> > **calculate** output percent uncertainties
> > **load** output variables
>
> **else**
>
> > **; LWC retrieval not run**
> > **load** output variables with error values and set status flags
>
> **end-if** (running LWC retrieval)

end-for (loop over profiles)

calculate metadata statistics

end 2B-LWC-RVOD

start 2B-IWC-RVOD

get orbit of 2B-GEOPROF data (CPR cloud mask, radar reflectivity)

get orbit of 2B-CLDCLASS data (cloud scenario)

get orbit of 2B-TAU data (visible optical depth)

for-each 2B-GEOPROF vertical profile

   convert bit flags to integer values
   determine if IWC retrieval will be run (known & valid cloud scenario, cloud present, Z physical)
   if running IWC retrieval (RVOD if optical depth available—RO otherwise)

      determine size of state vector
      assign *a priori* D_g, N_T, and ω values and uncertainties
      set $\mathbf{y}$ vector (2B-GEOPROF) using condensed profile retaining cloudy bins only
      set $\mathbf{S_a}$, $\mathbf{S_a^{-1}}$, $\mathbf{S_\epsilon}$ matrices
      repeat

         calculate $\mathbf{K}$, $\mathbf{S_y}$, $\mathbf{D_y}$ matrices
         calculate $\mathbf{F}$ (forward-model) vector
         calculate $\mathbf{S_y^{-1}}$, $\mathbf{S_x^{-1}}$, $\mathbf{S_x}$ matrices
         calculate new state vector $\hat{\mathbf{x}}$
         calculate $\Delta\hat{\mathbf{x}}$, convergence test
         if more than 15 iterations, reject

      end-repeat until convergetest $< 0.01n$
      calculate r_e, IWC, IWP
      calculate χ^2 and $\mathbf{A}$
      calculate retrieval uncertainties
      calculate output percent uncertainties
      load output variables

   else

      ; IWC retrieval not run
      load output variables with error values and set status flags

   end-if (running IWC retrieval)

end-for (loop over profiles)

calculate metadata statistics

end 2B-IWC-RVOD

start 2B-CWC-RVOD

get orbit of 2B-GEOPROF data

get orbit of 2B-LWC-RVOD data

get orbit of 2B-IWC-RVOD data

get orbit of ECMWF-AUX data

calculate ice phase fraction from temperature for all cloudy bins

calculate liquid phase fraction from temperature for all cloudy bins

map liquid properties into composite profiles, scaling bins by liquid phase fraction

map ice properties into composite profiles, scaling bins by ice phase fraction

calculate revised ice and liquid water path from composite profiles

copy error fill values into composite profiles

add error codes for other error conditions

set flags in status variable

write metadata statistics to text file

end 2B-CWC-RVOD

7 Data Product Output Format

The 2B-CWC-RVOD data product includes swath data and metadata in an HDF-EOS formatted file. Users are directed to the 2B-CWC-RVOD Interface Control Document for a full description of the data and metadata fields contained in the product. Scale factors used in converting file values into science data values are included in the file as HDF variable attributes. Users are encouraged to read scale factors directly from the file (rather than from written documentation), because the scale factors may change.

8 Changes since version 5.0

The following list summarizes the changes since version 5.0 of the 2B-CWC-RVOD product (used for internal testing only—never released):

- Liquid Water Content

  - Number concentration and width parameter now allowed to vary with altitude
  - Change scale factor of some HDF variables
  - Change model of radar uncertainty
  - Perform RO retrieval for profiles with no optical depth from 2B-TAU

- Ice Water Content

  - Change from modified gamma to lognormal size distribution
  - Number concentration and width parameter now allowed to vary with altitude
  - Change scale factor of some HDF variables
  - Change to temperature-based selection of *a priori* values
  - Omit cloudy bins warmer than $+1°C$ (because no *a priori* values apply)
  - Change parameterization of f_{Mie} ratio
  - Change model of radar uncertainty
  - Perform RO retrieval for profiles with no optical depth from 2B-TAU

- Cloud Water Content

  - Report profiles of all three size distribution parameters for both ice and liquid
  - Ice retrieval now uses lognormal distribution
  - Change scale factor of some HDF variables

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